ECON 133 Global Inequality and Growth
Section 10: Models of the wealth distribution

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April 5, 2022

How can we explain wealth distribution? Three main models:

1 The precautionary saving model

Theory: Individuals save to insure themselves for future risk. Individuals have uncertainty about their future earnings, and therefore save (i.e. consume less than their earnings) as a means of insuring their utility against a bad state of the world. Mathematically, individuals maximize an expected lifetime utility that weighs perceived future probabilities of being in good and bad states of earnings.

Possible interpretation: the richer you are, the less need to insure against labor income risk (e.g., the richer you are the less likely you are to be unemployed, so the less likely you will realize a bad state of the world). This interpretation would imply that the savings rate will fall with income and that wealth will be **more** equally distributed than income.

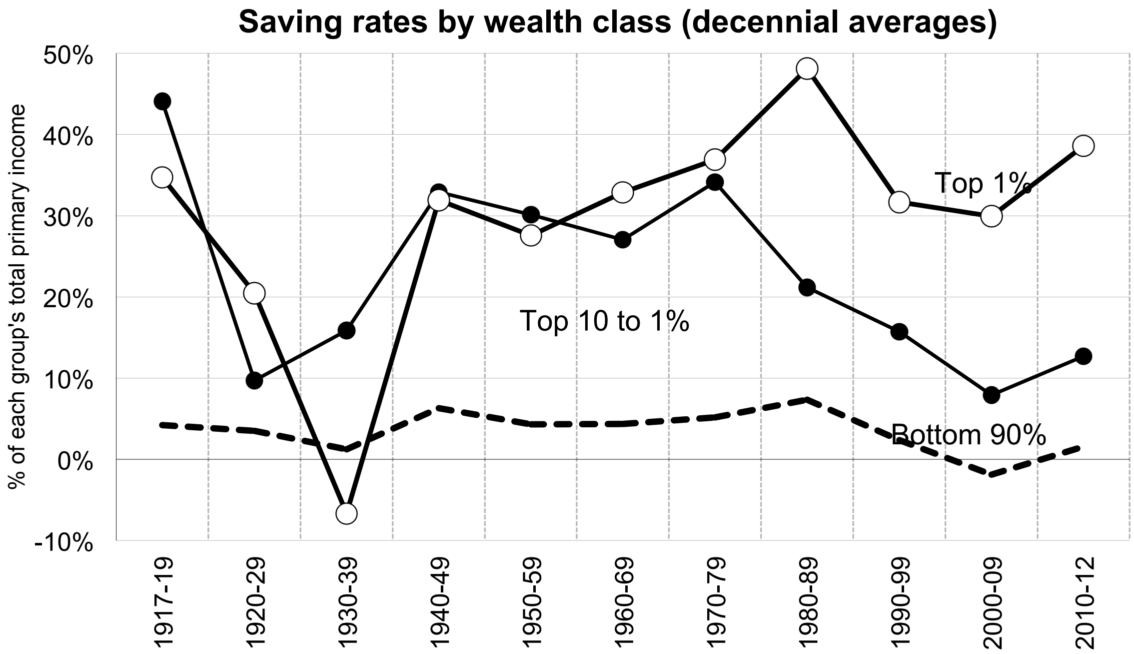
This theory is closely related to the Permanent Income Hypothesis: that individuals fully internalize their future expected earnings and therefore (in the presence of perfect financial markets) consume smoothly throughout their lives, regardless of their actual present earnings.

Caveats:

- People have other types of income that can factor into savings in ways that the model doesn't account for.
- Do people behave as expected utility maximizers?

*These notes borrow from past notes by José Díaz, Margie Lauter, Cristóbal Otero, All mistakes are my own.

What do the data say?



The rich save more as a fraction of their income, except in the 1930s when there was large dis-saving through corporations. NB: The average private saving rate has been 9.8% over 1913-2013.

Empirically we can see that the rich have a higher savings rate than the poor. The savings rate of the poor (e.g. bottom 90%) has decreased since 1975.

2 The life-cycle model

Theory: individuals save for retirement. Let's create a mathematical environment where all individuals live for Individuals live for L years, during which they work for N years and then retire.

Annual labor income can be expressed as:

$$\bar{Y} = \begin{cases} \frac{Y}{N} & \text{while working} \\ 0 & \text{when retired.} \end{cases}$$

Assuming individuals die with 0 wealth (i.e. no inheritance), individuals will smooth their consumption (C) over their whole lives. This means that during the working period of their lives, they will save part of their labor income ($S = \bar{Y} - C$), and dissave during retirement. Assume that individuals do not temporally discount utility.

Assume no growth ($n = g = r = 0$, i.e. capital is a pure storage technology and has no productive use).

Annual consumption: $C = \frac{N}{L} \cdot \bar{Y}$

Annual saving during working age: $S = \bar{Y} - C = \left(\frac{L-N}{L}\right) \cdot \bar{Y}$

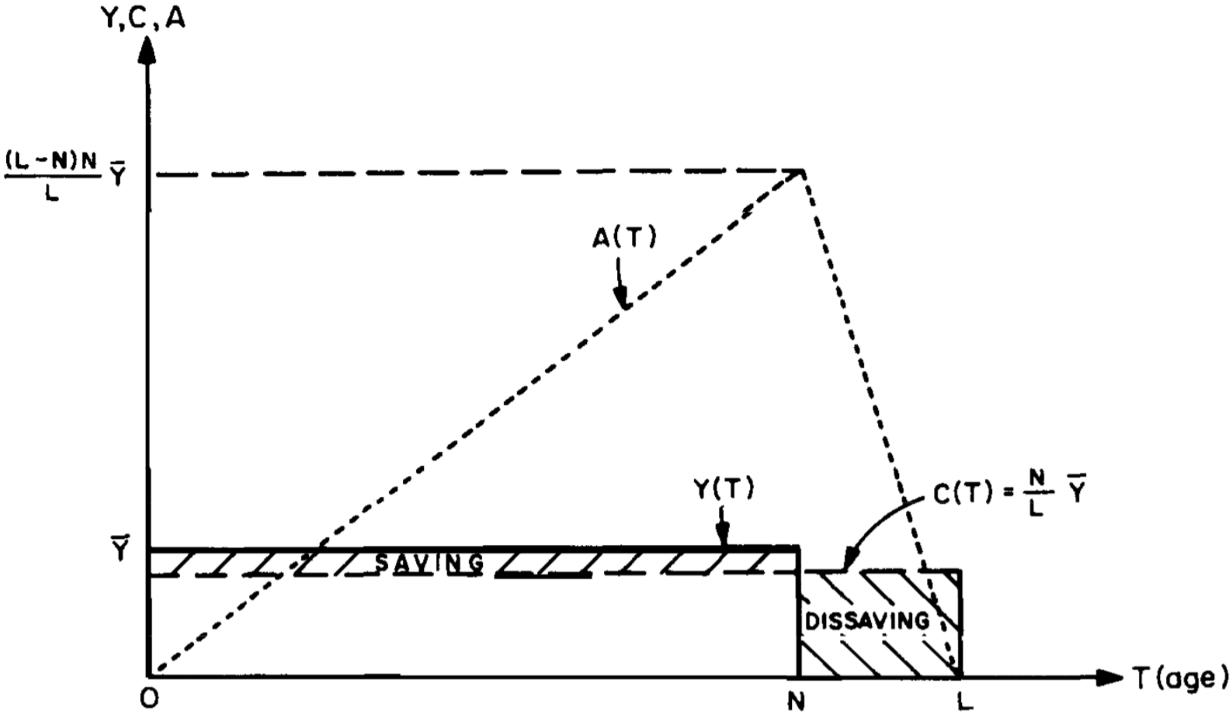
Annual dissaving after retirement: $S = -\frac{N}{L} \cdot \bar{Y}$ (the same as annual consumption during working life)

Wealth is simply annual saving times the number of years the individual saved (which is N). Wealth is denoted as "A" to be consistent with Modigliani's notation in the graph below, but you can think of it as the same wealth W we've always talked about in class. Wealth upon retirement is:

$$\begin{aligned} A &= N \cdot S \\ &= N \cdot \frac{(L-N)}{L} \cdot \bar{Y}. \end{aligned}$$

For a retired individual of age $X > N$, wealth can be expressed as:

$$\begin{aligned} A &= N \cdot S - (X-N) \cdot C \\ &= S - (X-N) \cdot \frac{N}{L} \cdot \bar{Y}. \end{aligned}$$



INCOME, CONSUMPTION, SAVING AND WEALTH AS A FUNCTION OF AGE

Source: Modigliani (1985)

We know that the wealth to income ratio in this model is:

$$\Rightarrow \frac{W}{Y} = \frac{1}{2}(L-N)$$

This is the Modigliani triangle formula: the aggregate wealth-income ratio equals one-half of retirement length.

Example: if retirement length is $L - N = 10$ years, then $\beta = \frac{10}{2} = 500\%$. Let's show it.

To simplify things, assume there is one individual per age bin in the economy.

Aggregate wealth, W, is:

$$
\begin{aligned}
W &= \text{Area of the Modigliani's triangle} \\
&= \frac{1}{2} \times \textit{base} \times \textit{height} \\
&= \frac{1}{2} \times L \times N \frac{(L-N)}{L} \bar{Y} \\
&= \frac{1}{2} N(L-N)\bar{Y}
\end{aligned}
$$

Aggregate income is simply average income times the number of individual in working age (between 0 and N):

$$
Y = N\bar{Y}
$$

Wealth to income ratio is:

$$
\begin{aligned}
\Rightarrow \frac{W}{Y} &= \frac{\frac{1}{2}N(L-N)\bar{Y}}{N\bar{Y}} \\
&= \frac{1}{2}(L-N)
\end{aligned}
$$

Caveats to the Modigliani triangle formula and framework (as presented here):

1. Wealth-income ratio depends only on demographics (i.e. $L - N$, the length of retirement).
2. Wealth inequality is as unequally distributed as labor income.
3. For a given age, everyone has the same amount of wealth.
4. Pure life-cycle motives (i.e. no bequest, where savings is only done for retirement). In fact, some of aggregate wealth comes from inherited wealth
5. People only rely on their savings during retirement. In reality, there is social security
6. The model assumes that people die with no wealth

Upshot: maybe we can accommodate some of these critiques

What does the data say? We don't actually observes a ton of life-cycle wealth pertaining to retirement (e.g. pension funds are at most 100-150% of income). In other words, only accounting for retirement savings, this model generates too little wealth inequality relative to what we see in reality – wealth is much more concentrated at the top than income.

Other predictions? E.g. timing of when individuals have max wealth?

3 Dynamic Random shocks models

None of the previous models can explain the existence of millionaires and billionaires. Without going into the math, let's describe the heuristics of a model that can address this gap.

In a world with different types of shocks—e.g. to

- r
- fertility rates
- saving tastes,

under a certain number of assumptions omitted here wealth converges to a steady-state distribution that has the following properties:

1. Wealth follows a Pareto law at the top of the distribution
2. The Pareto coefficient a depends on taste shocks s_{ti}
3. The higher the variance of shocks, the lower a
4. $a \to 1$ (and thus wealth inequality tends to infinity) if the variance of shocks goes to infinity, and $a \to \infty$ if the variance goes to zero

These kinds of models make structural assumptions on how wealth and income accumulate and interact as well as how shocks occur and influence present and future outcomes.
In the data: These kinds of complexity are sufficient to account for observed wealth concentration.